# Probabilistic modeling of RNA-Seq data 

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## RNA-Seq

Reads


Isoforms
$\square$
$\square$

Mapped reads


## RNA-Seq: mapping to genes

Reads


Genes


## How many reads are ambiguously mapped?

250000 RNA-Seq reads from Drosophila melanogaster

| Reference | \# of hits | reads |  |
| :--- | :--- | ---: | ---: |
| Genome | $>1$ hit | 13657 | $5.5 \%$ |
| Genome | $>10$ hits | 1638 | $0.7 \%$ |
| Transcriptome | $>1$ hit | 139410 | $55.8 \%$ |
| Transcriptome | $>10$ hits | 6197 | $2.5 \%$ |

## Using multireads

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$$
p(\tau \mid R)
$$

## Using the distribution

$$
\begin{aligned}
& p\left(\tau_{i}<\tau_{i}^{\prime} \mid R, R^{\prime}\right)=\iint \mathbb{I}\left(\tau_{i}<\tau_{i}^{\prime}\right) p(\tau \mid R) p\left(\tau^{\prime} \mid R^{\prime}\right) d \tau d \tau^{\prime} \\
& E\left(\tau_{i}-\tau_{i}^{\prime} \mid R, R^{\prime}\right)=\iint\left(\tau_{i}-\tau_{i}^{\prime}\right) p(\tau \mid R) p\left(\tau^{\prime} \mid R^{\prime}\right) d \tau d \tau^{\prime}
\end{aligned}
$$

## Bayes' theorem

$$
p(\tau \mid R)=p(R \mid \tau) \cdot \frac{p(\tau)}{p(R)}
$$

## Generative model



- $\tau$ - isoform expression levels
- $G_{n}$ - isoform of read $n$
- $S_{n}$ - start position of read $n$ in the isoform
- $R_{n}$ - nucleotide sequence of read $n$


## How to compute the integral?

Grid: $\tau^{(1)}, \tau^{(2)}, \ldots, \tau^{(n)} \sim p(\tau \mid R)$ uniformly distributed

$$
\int f(\tau) p(\tau \mid R) d \tau \approx \sum_{i=1}^{n} f\left(\tau^{(i)}\right) p\left(\tau^{(i)} \mid R\right) \Delta \tau
$$

1 TPM resolution per isoform, 30000 isoforms $\Rightarrow$ $\approx 10^{60000}$ points

## Better representation

$$
\begin{gathered}
\tau^{(1)}, \tau^{(2)}, \ldots, \tau^{(n)} \sim p(\tau \mid R) \\
\int f(\tau) p(\tau \mid R) d \tau \approx \frac{1}{n} \sum_{i=1}^{n} f\left(\tau^{(i)}\right)
\end{gathered}
$$

Very easy to compute with!

## How to geneate samples?

It is a hard problem in general.
Key idea: Markov chain Monte Carlo (MCMC)

## Further reading

1. RNA-Seq gene expression estimation with read mapping uncertainty (Li et al., 2009)
2. RSEM: accurate transcript quantification from RNA-Seq data with or without a reference genome (Li et al., 2011)
3. RNA-Seq data analysis through expectationmaximization (my blog post, https://ro-che.info/articles/2017-01-29-rsem)

## Conclusions

1. Prefer isoforms over genes
2. Probabilistic modeling accounts for uncertainty
3. MCMC makes probabilistic modeling viable
