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Our clients are investors. What are their needs?



Investor #1:

- is sure that Microsoft stock will go up in a year
- wants to make money on his prediction
- buys Microsoft stock
- sells it in one year, earning (or losing) the difference in prices



Investor #2:

- is sure that Microsoft stock will go up in a year
- wants to make money on his prediction
- concerned about his possible losses
- \blacktriangleright \Rightarrow does not buy Microsoft stock



What can we offer?

- ▶ if the stock goes up, we pay the difference in prices
- otherwise, no payment happens

Mathematically

$$payout = \max(S_1 - S_0, 0)$$



$$payout = \max(S_1 - S_0, 0)$$

In Haskell:

payout :: Double -> Double -> Double payout s0 s1 = max (s1 - s0) 0



```
payout :: Market -> Double
payout market =
    let s0 = observe market "26-05-2011" "MSFT"
    s1 = observe market "26-05-2012" "MSFT"
    in max (s1 - s0) 0
```



Good:

- Unambiguously specifies the contract
- Allows to calculate the payoff when the contract expires
 Bad:
 - Can't be executed before the contract expires



Things we want to know about the contract:

- Set of observation dates
- Set of underlying securities (eg. stocks)
- Points of discontinuities of the payoff function

► ...



Solution:

- parse the program;
- analyse abstract syntax tree and extract the necessary information



How to represent abstract syntax tree?

Haskell's answer: Algebraic Data Types

- combine unions and structs from C
- resemble Backus-Naur form for the grammar



data Expr = EAdd Expr Expr

- | ESub Expr Expr
- | EMax Expr Expr
- | EConst Double
- | EAsset String
- | EDate String
- EObserve Expr Expr



Representation of our contract:

EMax

(ESub (EObserve (EDate "26-05-2012") (EAsset "MSFT")) (EObserve (EDate "26-05-2011") (EAsset "MSFT")) (EConst 0)



Extract stored values using pattern matching

```
listOfDates :: Expr -> [Date]
listOfDates e =
    case e of
        EDate date -> [date]
        EAsset asset -> []
        EConst x -> []
        EAdd e1 e2 -> listOfDates e1 ++ listOfDates e2
        ESub e1 e2 -> listOfDates e1 ++ listOfDates e2
        EMax e1 e2 -> listOfDates e1 ++ listOfDates e2
        . . .
```



Parsing is awkward. Can we avoid it?



```
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```
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EMax
        (ESub
            (EObserve (EDate "26-05-2012") (EAsset "MSFT"))
            (EObserve (EDate "26-05-2011") (EAsset "MSFT"))
        (EConst 0)
```



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        (EConst 0)
```

Yes! Redefine the functions to generate the syntax tree.



Redefine the functions to generate the syntax tree.

max e1 e2 = EMax e1 e2 e1 + e2 = EAdd e1 e2 e1 - e2 = ESub e1 e2

observe date asset = EObserve date asset



We can even overload numeric and string literals!

instance Fractional Expr where
 fromRational x = EConst (fromRational x)



Investor #3:

- is concerned about possible fluctuations
- wants to average the observations



```
payout =
    let s0 = observe "26-05-2011" "MSFT"
```

s1 = observe "26-05-2012" "MSFT"
s2 = observe "26-06-2012" "MSFT"
s3 = observe "26-07-2012" "MSFT"
avg = (s1 + s2 + s3)/3

in max (avg - s0) 0



```
payout =
    let s0 = observe "26-05-2011" "MSFT"
    s1 = observe "26-05-2012" "MSFT"
    s2 = observe "26-06-2012" "MSFT"
    s3 = observe "26-07-2012" "MSFT"
    avg = (s1 + s2 + s3)/3
```

in max (avg - s0) 0

Good programmers don't write code like this!



foldl :: $(a \rightarrow b \rightarrow a) \rightarrow a \rightarrow [b] \rightarrow a$



```
foldl :: (a \rightarrow b \rightarrow a) \rightarrow a \rightarrow [b] \rightarrow a
```

sum :: [Double] -> Double
sum list = foldl (+) 0 list



```
foldl :: (a \rightarrow b \rightarrow a) \rightarrow a \rightarrow [b] \rightarrow a
```

```
sum :: [Double] -> Double
sum list = foldl (+) 0 list
```

```
length :: [Double] -> Double
length list = foldl (\acc x -> acc + 1) 0 list
```



```
foldl :: (a \rightarrow b \rightarrow a) \rightarrow a \rightarrow [b] \rightarrow a
```

```
sum :: [Expr] -> Expr
sum list = foldl (+) 0 list
```

```
length :: [Expr] -> Expr
length list = foldl (\acc x -> acc + 1) 0 list
```





Task: print a mathematical formula that describes the contract Large ASTs lead to large formulas



Task: print a mathematical formula that describes the contract

Large ASTs lead to large formulas

Solution: make fold a part of our language!



data Expr = ...
| EFoldl Function2 Expr [Expr]



```
data Expr = ...
| EVar VarId
| EFoldl Function2 Expr [Expr]
```

type Function2 = (VarId, VarId, Expr)

```
type VarId = Int
```



foldl f a xs = EFoldl (lambdaToFunction2 f) a xs

lambdaToFunction2 :: (Expr -> Expr -> Expr) -> Function2 lambdaToFunction2 f = ?



foldl f a xs = EFoldl (lambdaToFunction2 f) a xs

lambdaToFunction2 :: (Expr -> Expr -> Expr) -> Function2
lambdaToFunction2 f =
 (EVar 0, EVar 1, f (EVar 0) (EVar 1))



foldl f a xs = EFoldl (lambdaToFunction2 f) a xs

lambdaToFunction2 :: (Expr -> Expr -> Expr) -> Function2
lambdaToFunction2 f =
 (EVar 0, EVar 1, f (EVar 0) (EVar 1))

(plus extra care to avoid free variable capture)



observe (EAsset asset) (EDate date)

Can you spot the error?



observe (EAsset asset) (EDate date)

Can you spot the error?

Correct form:

observe (EDate date) (EAsset asset)

Can the compiler catch this?



```
newtype Date = Date Expr
newtype Asset = Asset Expr
newtype Number = Number Expr
```

```
observe :: Date -> Asset -> Number
observe (Date date) (Asset asset) =
    Number (EObserve date asset)
```



FPF = Functional Payout Framework

Language + Set of tools (backends)

- Generate mathematical formulas
- Extract sets of dates and assets
- Analyse for discontinuities
- Generate C code for Monte-Carlo simulation
- ... and more

Frankau et al. "Going functional on exotic trades"



Using Haskell for a domain-specific language:

- higher-order functions
- no need in parsing
- strong static type system
- type inference
- rich overloading

All for free!



Why work at Barclays Capital?

- real-world usage of functional programming
- work among smart people
- solve interesting problems
- get immediate feedback on your work

Send your CV to Roman.Cheplyaka@BarclaysCapital.com

